

A Cost Comparative Analysis for Economic Load Dispatch Problems Using Modern Optimization Techniques

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Received 16 December; accepted 18 January; published online 01 February; printed 16 February 2013

ABSTRACT

The Economic Load Dispatch (ELD) problems are the major consideration in electric power generation systems in order to reduce the fuel cost their by reducing the total cost for the generation of electric power. This paper presents an Efficient and Reliable Firefly Algorithm (FA) for solving ELD Problem. The main objective is to minimize the total fuel cost of the generating units having quadratic cost characteristics subjected to limits on generator True power output & transmission losses. It has been achieved by using optimization techniques such as dynamic programming, integer programming, and mixed-integer non-linear programming. On the other hand, a broad class of Meta heuristics has been developed for reliability-redundancy optimization. Recently, a new meta-heuristics called Firefly Algorithm (FA) algorithm has emerged. The FA is a stochastic Meta heuristic approach based on the idealized behaviour of the flashing characteristics of fireflies. This paper presents an application of the Firefly Algorithm (FA) to ELD for different Test Case system. ELD is applied and compared its solution quality and computation efficiency to Simulated Annealing (SA), Genetic algorithm (GA), Differential Evolution (DE), Particle swarm optimization (PSO), Artificial Bee Colony optimization (ABC), and Biogeography-Based Optimization (BBO) optimization techniques. The simulation results show that the proposed algorithm outperforms previous optimization methods.

Keywords: Firefly Algorithm; Economic Load dispatch; Genetic algorithm; Particle swarm optimization; Artificial Bee Colony optimization; Biogeography-Based Optimization.

1. INTRODUCTION

Electrical power industry restructuring has created highly vibrant and competitive market that altered many aspects of the power industry. In this changed scenario, scarcity of energy resources, increasing power generation cost, environment concern, ever growing demand for electrical energy necessitate optimal economic dispatch. Economic Load Dispatch (ELD) is one of the important optimization problems in power systems that have the objective of dividing the power demand among the online generators economically while satisfying various constraints. Since the cost of the power generation is exorbitant, an optimum dispatch saves a considerable amount of money. Optimal generation dispatch is one of the most important problems in power system engineering, being a technique commonly used by operators in every day system operation. Optimal generation seeks to allocate the real and reactive power throughout power system obtaining optimal operating state that reduces cost and improves overall system efficiency. The ELD problem reduces system cost by allocating the real power among online generating units. In the ELD problem the classical formulation presents deficiencies due to simplicity of models. Here, the power system modelled through the power balance equation and generators are modelled with smooth quadratic cost functions and generator output constraints.

To improve power system studies, new models are continuously being developed that result in a more efficient system operation. Cost functions that consider valve point loadings, fuel switching, and prohibited operating zones as well as constraints that provide more accurate representation of system such as: emission, ramp rate limits, line flow limits, spinning reserve requirement and system voltage profile. The improved models generally increase the level of complexity of the optimization problem due to the non-linearity associated with them.

Traditional algorithms like lambda iteration, base point participation factor, gradient method, and Newton method can solve the ELD problems effectively if and only if the fuel-cost curves of the generating units are piece-wise linear and monotonically increasing. The basic ELD considers the power balance constraint apart from the generating capacity limits. However, a practical ELD must take ramp rate limits, prohibited operating zones, valve point effects, and multi fuel options into consideration to provide the completeness for the ELD formulation. The resulting ELD is a non-convex optimization problem, which is a challenging one and cannot be solved by the traditional methods. Practical ELD problems have nonlinear, non-convex type objective function with intense equality and inequality constraints. Recent advances in computation and the search for better results of complex optimization problems have fomented the development of techniques known as Evolutionary Algorithms. Evolutionary Algorithms are stochastic based optimization techniques that search for the solution of problems using a simplified model of the evolutionary process. These algorithms provide an alternative for obtaining global optimal solutions, especially in the presence of non-continuous, non-convex, highly solution spaces. These algorithms are population based techniques which explore the solution space randomly by using several candidate solutions instead of the single solution estimate used by many classical techniques. The success of evolutionary algorithms lies in the capability of finding solutions with random exploration of the feasible region rather than exploring the complete region. This result in a faster optimization process with lesser computational resources while maintaining the capability of finding global optima. The conventional optimization methods are not able to solve such problems as due to local optimum solution convergence. Meta-heuristic optimization techniques especially Genetic Algorithms (GA), Particle Swarm Optimization (PSO) and Differential Evaluation (DE) gained an incredible recognition as the solution algorithm for such type of ELD problems in last decade.

2. ECONOMIC LOAD DISPATCH (ELD) PROBLEM

The classical ELD problem is an optimization problem that determines the power output of each online generator that will result in a least cost system operating state. The objective of the classical ELD is to minimize the total system cost where the total system cost is a function composed by the sum of the cost functions of each online generator. This power allocation is done considering system balance between generation and loads, and feasible regions of operation for each generating unit.

2.1. Objective Function and Constraints

The objective of the classical ELD is to minimize the total system cost by adjusting the power output of each of the generators connected to the grid. The total system cost is modelled as the sum of the cost function of each generator.

The basic ELD problem can be described mathematically as a minimization of problem.

$$\text{Minimize} \quad \sum_{i=1}^n F_i(P_i) \quad \dots(1)$$

Where $F_i(P_i)$ is the fuel cost equation of the 'i'th plant. It is the variation of fuel cost (\$ or Rupees) with generated Power (MW). Normally it is expressed as a quadratic equation as

$$F_i(P_i) = a_i P_i^2 + b_i P_i + c_i \quad \dots (2)$$

The total fuel cost is to be minimized subject to the following constraints.

$$\sum_{i=1}^n P_i = P_d + P_l \quad \dots (3)$$

$$P_l = \sum_i^n \sum_j^j P_i B_{ij} P_j \quad \dots (4)$$

By Lagrangian multipliers method and Kuhn tucker conditions and the following conditions for optimality can be obtained

$$2a_i P_i + b_i = \lambda (1 - 2 \sum_{i,j=1}^n B_{ij}) \quad \dots(5)$$

$$\sum_{i=1}^n P_i = P_d + P_l \quad \dots (6)$$

$$P_l = \sum_i^n \sum_j^j P_i B_{ij} P_j \quad \dots (7)$$

$$\dots (P_i^{\min} \leq P_i \leq P_i^{\max}) \quad 8)$$

The non-linear equations and inequalities are solved by initializing the procedure allocate lower limit of each plant as generation, evaluate the transmission loss and incremental loss coefficients and update the demand.

$$P_i = P_i^{\min}, X_i = 1 - \sum_{i,j}^n B_{ij} P_j, P_d^{\text{new}} = P_d + P_l^{\text{old}} \quad \dots (9)$$

By substituting the incremental cost coefficients and solving the set of linear equations the incremental fuel cost is determined.

$$\lambda = \frac{\sum_i^n \frac{b_i}{2a_i}}{P_d^{\text{new}} + \frac{X_i}{2a_i}} \quad \dots (10)$$

For determining the power allocation of each plant by

$$P_i^{\text{new}} = \frac{\frac{\lambda - b_i}{2a_i}}{\frac{X_i}{2a_i}} \quad \dots (11)$$

If plant violates its limits it should be fixed to that limit and the remaining plants only be considered for next iteration, and finally check for convergence.

3. THE FIREFLY ALGORITHM

The Firefly Algorithm [FA] is a Meta heuristic, nature-inspired, optimization algorithm which is based on the social flashing behavior of fireflies, or lighting bugs, in the summer sky in the tropical temperature regions. It was developed by Dr. Xin-She Yang at Cambridge University in 2007, and it is based on the swarm behavior such as fish, insects, or bird schooling in nature. In particular, although the firefly algorithm has many similarities with other algorithms which are based on the so-called swarm intelligence, such as the famous Particle Swarm Optimization [PSO] and Artificial Bee Colony optimization [ABC] algorithms it is indeed much simpler both in concept and implementation. Furthermore, according to recent bibliography, the algorithm is very efficient and can outperform other conventional algorithms, such as genetic algorithms, for solving many optimization problems; a fact that has been justified in a recent research, where the statistical performance of the firefly algorithm was measured against other well-known optimization algorithms using various standard stochastic test functions. Its main advantage is the fact that it uses mainly real random numbers, and it is based on the global communication among the swarming particles [i.e., the fireflies], and as a result, it seems more effective in optimization such as the ELD problem in our case. The FA has three particular idealized rules which are based on some of the major flashing characteristics of real fireflies. These are the following:

- All fireflies are unisex, and they will move towards more attractive and brighter ones regardless their sex.
- The degree of attractiveness of a firefly is proportional to its brightness which decreases as the distance from the other firefly increases due to the fact that the air absorbs light. If there is not a brighter or more attractive firefly than a particular one, it will then move randomly.
- The brightness or light intensity of a firefly is determined by the value of the objective function of a given problem. For maximization problems, the light intensity is proportional to the value of the objective function.

3.1. Attractiveness

In the FA, the form of attractiveness function of a firefly is the following monotonically decreasing function:

$$\beta(r) = \beta_0 \exp(-\gamma r^m) \quad \text{with } m \geq 1 \quad (\text{model 1})$$

Where, r is the distance between any two fireflies, β_0 is the initial attractiveness at $r=0$, and γ is an absorption coefficient which controls the decrease of the light intensity.

3.2. Distance

The distance between any two fireflies i and j , at positions x_i and x_j , respectively, can be defined as a Cartesian or Euclidean distance as follows:

$$r_{ij} = \|x_i - x_j\| = \sqrt{\sum_{k=1}^d (x_{i,k} - x_{j,k})^2} \quad (\text{model 2})$$

Where $x_{i,k}$ is the k^{th} component of the spatial coordinate x_i of the i^{th} firefly and d is the number of dimensions we have, for $d=2$, we have

$$r_{ij} = ((x_i - x_j)^2 + (y_i - y_j)^2) \quad (\text{model 3})$$

However, the calculation of distance r can also be defined using other distance metrics, based on the nature of the problem, such as Manhattan distance or Mahalanobis distance.

3.3. Movement

The movement of a firefly i which is attracted by a more attractive (i.e., brighter) firefly j is given by the following equation:

$$x_i = x_i + \beta_0 \exp(-\gamma r_{ij}^2) * (x_j - x_i) + \alpha * (\text{rand} - \frac{1}{2}) \quad (\text{model 4})$$

Where the first term is the current position of a firefly, the second term is used for considering a firefly's attractiveness to light intensity seen by adjacent fireflies, and the third term is used for the random movement of a firefly in case there are not any brighter ones. The coefficient α is a randomization parameter determined by the problem of interest, while rand is a random number generator uniformly distributed in the space $(0, 1)$. As we will see in this implementation of the algorithm, we will use $\beta_0 = 1.0$, $\alpha \in [0, 1]$ and the attractiveness or absorption coefficient $\gamma \in [1.0]$, which guarantees a quick convergence of the algorithm to the optimal solution.

3.4. Convergence and Asymptotic Behaviour

The convergence of the algorithm is achieved for any large number of fireflies (n) if $n \gg m$, where m is the number of local optima of an optimization problem [1, 3]. In this case, the initial location of n fireflies is distributed uniformly in the entire search space. The convergence of the algorithm into all the local and global optima is achieved, as the iterations of the algorithm continue, by comparing the best solutions of each iteration with these optima. However, it is under research a formal proof of the convergence of the algorithm and particularly that the algorithm will approach global optima when $n \rightarrow \infty$ and $t \gg 1$. In practice, the algorithm converges very quickly in less than 80 iterations and less than 50 fireflies, as it is demonstrated in several research papers using some standard test functions. Indeed, the appropriate choice of the number of iterations together with the γ , β , α , and n parameters highly depends on the nature of the given optimization problem as this affects the convergence of the algorithm and the efficient find of both local and global optima. Note that the firefly algorithm has computational complexity of $O(n)^2$ where n is the population of fireflies. The larger population size becomes the greater the computational time.

3.5. Special Cases

There are two important special cases of the firefly algorithm based on the absorption coefficient γ ; that is, when $\gamma \rightarrow 0$ and $\gamma \rightarrow \infty$. When $\gamma \rightarrow 0$, the attractiveness coefficient is constant $\beta = \beta_0$, and the light intensity does not decrease as the distance r between two fireflies increases. Therefore, as the light of a firefly can be seen anywhere, a single local or global optimum can be easily reached. This limiting case corresponds to the standard Particle Swarm Optimization (PSO) algorithm. On the other hand, when $\gamma \rightarrow \infty$, the attractiveness coefficient is the Dirac delta function $\beta(r) \delta(r)$. In this limiting case, the attractiveness to light intensity is almost zero, and as a result, the fireflies cannot see each other, and they move completely randomly in a foggy place. Therefore, this method corresponds to a random search method.

3.6. Hybridization

In a recent bibliography, a new Meta heuristic algorithm has been developed and formulated based on the concept of hybridizing the firefly algorithm. In particular, the new Levy flight Firefly algorithm was developed by Dr. Xin-She Yang at Cambridge University in 2010 and it combines the firefly algorithm with the Levy flights as an efficient search strategy. It combines the three idealized rules of the firefly algorithm together with the characteristics of Levy flights which simulate the flight behavior of many animals and insects. In this algorithm, the form of the attractiveness function and the calculation of distance between two fireflies are the same as in firefly algorithm, but in the movement function, the random step length is a combination of the randomization parameter together with a Levy flight. In particular, the movement of a firefly is a random walk, where the step length is drawn by the Levy distribution.

4. THE FIREFLY ALGORITHM (FA) SOLUTION

In order to solve the ELD problem, we have implemented the FA in Matlab and it was run on a computer with an Intel Core2 Duo (1.8GHz) processor, 3GB RAM memory and MS Windows XP as an operating system. Mathematical calculations and comparisons can be done very quickly and effectively with Matlab and that is the reason that the proposed FA was implemented in Matlab programming environment. In this proposed method, we represent and associate each firefly with a valid power output (i.e., potential solution) encoded as a real number for each power generator unit, while the fuel cost objective i.e., the objective function of the problem is associated and represented by the light intensity of the fireflies.

In this simulation, the values of the control parameters are: $\alpha = 0.2$, $\gamma = 1.0$, $\beta_0 = 1.0$ and $n = 12$, and the maximum generation of fireflies (iterations) is 50. The values of the fuel cost, the power limits of each generator, the power loss coefficients, and the total power load demand are supplied as inputs to the firefly algorithm. The power output of each generator, the total system power, the fuel cost with/without transmission losses are considered as outputs of the proposed Firefly algorithm. Initially, the objective function of the given problem is formulated and it is associated with the light intensity of the swarm of the fireflies. The initial solution of the given problem is generated based on the mathematical formulation given below:

$$x_j = \text{rand} * (\text{upper-range} - \text{lower-range}) + \text{lower-range} \quad (\text{model } 5)$$

Where x_j is the new solution of j th firefly, that is, created, rand is a random number generator uniformly distributed in the space $[0, 1]$, while upper range and lower range are the upper range and lower range of the j th firefly (variable), respectively.

After the evaluation of the initial population/generation (i.e., solution), the firefly algorithm enters its main loop which represents the maximum number of generations of the fireflies. This is actually the termination criterion that needs to be satisfied for the termination of the loop.

The generation of a new solution (i.e., the movement of a firefly) of the given problem is made based on the following mathematical formulation:

$$x_i = x_i + \beta_0 * \exp(-\gamma * \sum_{j=1}^d (x_i - x_j)^2) * (x_i - x) + a * (\text{rand} - \frac{1}{2}) \quad (\text{model } 6)$$

Where x_i is the current solution of the i th firefly and x_j is the current (optimal) solution of j th firefly.

The values of the algorithm's control parameters is $\alpha = 0.2$, $\gamma = 1.0$, $\beta_0 = 1.0$, and rand is a random number which is uniformly distributed in the space $[0, 1]$. As we can see the distance between two fireflies is calculated using the Euclidean distance and the generation of a new solution is actually a sum of the current solution (x_i), the metric of the evaluation of the current solution based on the current optimal solution (Euclidean metric), and a random step/move of the algorithm. After the generation of the new solutions, we have to apply the generator capacity constraints so as the new solutions are within the given operational power ranges. To avoid such violation, a repair process is applied to each solution (firefly) in order to guarantee that the generated power outputs are feasible. P_k , P_k min and P_k max denote the current, the minimum, and the maximum power outputs of the i th unit, which is associated with the i th firefly. Finally, it is notable that for each generation (iteration), the swarm of 12 fireflies is ranked based on their light intensity, and the firefly with the maximum light intensity (i.e., the solution with the higher objective function value) is chosen as the brighter one (i.e., it is a potential optimal solution), while the others are updated. In the final iteration, the firefly with the brighter light intensity among the swarm of 12 fireflies is chosen as the brightest one which represents the optimal solution of the problem.

5. SIMULATION RESULTS AND DISCUSSION

To solve the ELD problem, we have implemented the FA in Matlab and it was run on a computer with an Intel Core2 Duo (1.8GHz) processor, 3GB RAM memory and MS Windows XP as an operating system. Mathematical calculations and

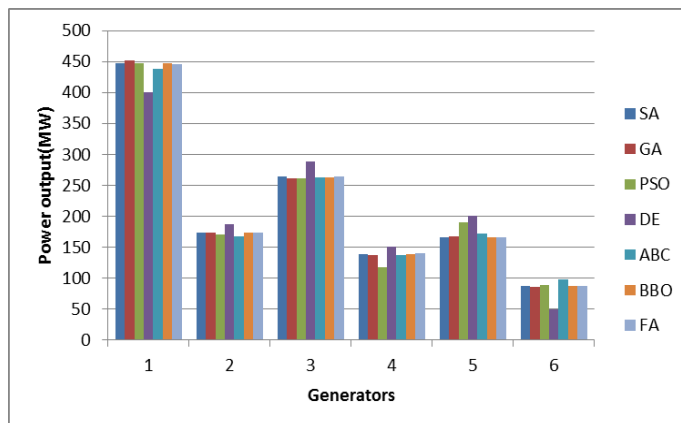


Figure 1

Generator outputs for various algorithms

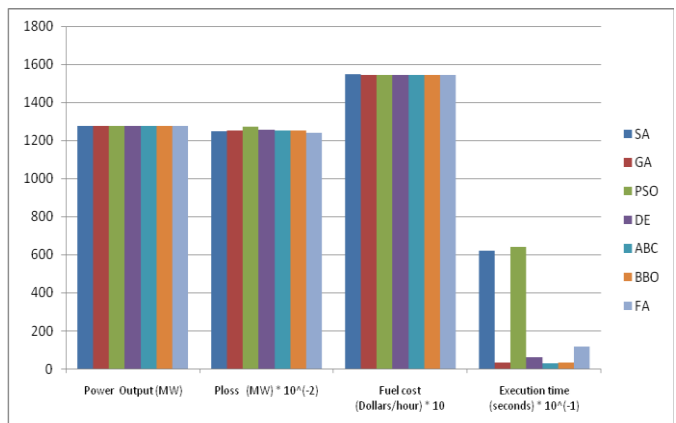


Figure 2

Comparison chart showing all the parameters

Table 1 Generating unit capacity and coefficients

Unit	P_{min}	P_{max}	a_i	b_i	c_i
1	100	500	240	7.0	0.0070
2	50	200	200	10.0	0.0095
3	80	300	220	8.5	0.0090
4	50	150	200	11.0	0.0090
5	50	200	220	10.5	0.0080
6	50	120	190	12.0	0.0075

Table 2 Comparison table for the best results

Sl.No	Description	SA	G.A	PSO	D.E	ABC	BBO	FA
1.	P_1 (MW)	447.08	451.97	447.50	400.00	438.65	447.39	445.08
2.	P_2 (MW)	173.18	173.16	170.52	186.55	167.90	173.24	173.08
3.	P_3 (MW)	263.92	261.16	261.90	289.00	262.82	263.32	264.42
4.	P_4 (MW)	139.06	136.85	116.91	150.00	136.77	138.00	139.59
5.	P_5 (MW)	165.58	166.70	190.41	200.00	171.76	165.41	166.02
6.	P_6 (MW)	86.63	85.68	88.49	50.00	97.67	87.80	87.21
7.	Power Output (MW)	1275.47	1275.52	1275.73	1275.55	1275.57	1275.50	1275.40
8.	P_{loss} (MW)	12.47	12.52	12.73	12.55	12.52	12.50	12.40
9.	Fuel cost (Dollars/hour)	15466.00	15458.00	15456.56	15452.00	15445.90	15443.09	15443.00
10.	Execution time(seconds)	62.02	3.18	64.09	6.20	2.82	3.02	11.52

comparisons can be done very quickly and effectively with Mat lab and that is the reason that the proposed Firefly algorithm was implemented in Matlab programming environment. Since the performance of the proposed algorithm sometimes depends on input parameters, they should be carefully chosen. After several runs, the following input control parameters are found to be best for optimal performance of the proposed algorithm.

In this proposed method, we represent and associate each firefly with a valid power output (i.e., potential solution) encoded as a real number for each power generator unit, while the fuel cost objective i.e., the objective function of the problem is associated and represented by the light intensity of the fireflies. In this simulation, the values of the control parameters are: $\alpha = 0.2$, $\gamma = 1.0$, $\beta_0 = 1.0$ and $n = 12$, and the maximum generation of fireflies (iterations) is 50. The values of the fuel cost, the power limits of each generator, the power loss coefficients, and the total power load demand are supplied as inputs to the firefly algorithm. The power output of each generator, the total system power, the fuel cost with/without transmission losses are considered as outputs of the proposed Firefly algorithm. Initially, the objective function of the given problem is formulated and it is associated with the light intensity of the swarm of the fireflies.

The FA has been proposed for IEEE - 30 Bus with six generator test system in the references. This power system is connected through 41 transmission lines and the demand 1263MW. The input and the cost coefficient of IEEE - 30 Bus with six generator test system are given in table 1. In this system SA, GA, PSO, DE, ABC, BBO & FA Algorithms were used in ELD. In table 2, results obtained from proposed FA method has been compared with other methods. According to the result obtained using the FA for ELD is more advantageous than other Algorithms. Power System with six generator test system, if ELD is realized by using the FA and the total line loss is decreased.

6. CONCLUSION

The proposed FA to solve ELD problem by considering the practical constraints has been presented in this paper. From the comparison table it is observed that the proposed algorithm exhibits a comparative performance with respect to other population based techniques. It is clear from the results that Biogeography Based Optimization algorithm is capable of obtaining higher quality solution with better computation efficiency and stable convergence characteristic. The effectiveness of FA was demonstrated and tested. From the simulations, it can be seen that FA gave the best result of total cost minimization compared to the other methods. In future, the proposed FA can be used to solve ELD with considering the valve point effects, which is still in the progress of the research work.

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